

# Robust and Scalable Scheme to Generate Large-Scale Entanglement Webs

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We propose a robust and scalable scheme to generate an  $N$ -qubit  $W$  state among separated quantum nodes (cavity-QED systems) by using linear optics and postselections. The present scheme inherits the robustness of the Barrett-Kok scheme [Phys. Rev. A **71**, 060310(R) (2005)]. The scalability is also ensured in the sense that an arbitrarily large  $N$ -qubit  $W$  state can be generated with a quasi-polynomial overhead  $\sim 2^{O[(\log_2 N)^2]}$ . The process to breed the  $W$  states, which we introduce to achieve the scalability, is quite simple and efficient, and can be applied for other physical systems.

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*Introduction.*— So far tremendous efforts have been paid for experimental realizations of quantum information processing (QIP), and, for example, control of a few qubits has been performed in cavity QED, ion traps, etc. It, however, seems difficult to increase the number of qubits dramatically within a single physical system. In order to realize large-scale QIP, we have to develop a novel way to integrate individual physical systems scalably. Furthermore, for communication purposes, quantum information has to be shared among separated quantum nodes. To meet these requirements, *distributed* QIP, where stationary qubits are entangled by using flying qubits (photons), seems to be very promising [1–4]. A lot of protocols have been proposed so far for remote entangling operations and probabilistic two-qubit gates [5–7]. The Barrett-Kok scheme is particularly promising, since it is fully scalable and robust against experimental imperfections [6]. Experiments of the remote entangling operations (or probabilistic two-qubit gates) between separated qubits have also been done in both atomic ensembles [8] and trapped single atoms [9]. They are important ingredients for fault-tolerant distributed quantum computation [6, 10–12].

Multipartite entanglement is not only a key ingredient for quantum communication, but also an important clue to understand nature of quantum physics. There are a lot of classes of multipartite entanglement, for example, GHZ (Greenberger-Horne-Zeilinger) states [13], cluster states [14], and  $W$  states [15], which cannot be transformed each other under local operations and classical communication (LOCC). Among them, the  $W$  states,

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|100\cdots 0\rangle + |010\cdots 0\rangle \cdots + |000\cdots 1\rangle),$$

are quite robust in the sense that any pairs of qubits are still entangled, even if the rest of the qubits are discarded [16]. This web-like property is very fascinating as universal resources, i.e., *entanglement webs*, for quantum communication. There are several protocols, which use the  $W$  states for quantum key distribution, teleportation, leader election, and information splitting [17]. Further-

more, inevitable decoherence in sharing the  $W$  states can be counteracted by using a novel scheme of purification [18]. The preparation of the  $W$  states by using optics has been discussed so far extensively both theoretically and experimentally [19]. It has been also discussed in other systems, such as cavity QED and ion traps [20]. Nevertheless none of them seems to be fully scalable. That is, overhead required for sharing an  $N$ -qubit  $W$  state scales exponentially in the number of qubits  $N$ , or the  $W$  state is prepared in a single system, which cannot be used for quantum communication among separated quantum nodes.

In this paper, we develop a *robust and scalable* scheme to generate the  $N$ -qubit  $W$  state by using separated cavity-QED systems and linear optics. The present scheme is scalable in the sense that an arbitrarily large  $N$ -qubit  $W$  state can be generated among separated quantum nodes with only a quasi-polynomial overhead  $\sim 2^{O[(\log_2 N)^2]}$ . In the following, we first develop an efficient way to generate the four-qubit  $W$  state  $|W_4\rangle$  by following the concept of the Barrett-Kok scheme [6], which is quite robust against the experimental imperfections. The success probability to obtain the  $|W_4\rangle$  is significantly high to be  $1/2$ . Then, by using the four-qubit  $W$  states as *seeds*, we can *breed* an arbitrarily large  $W$  state in an economical way, where the two  $|W_N\rangle$ 's are converted to one  $|W_{2(N-1)}\rangle$  probabilistically by accessing only two qubits. In contrast to classical webs, where a local connection does not result in a global web structure, this property of entanglement webs is a genuine quantum phenomenon. Even if the conversion fails, the two  $|W_{N-1}\rangle$ 's are left and can be recycled. This breeding method is quite simple and economical, and can be applied to other physical systems, such as polarization qubits in optics [19].

*Four-qubit  $W$  state (seeding).*— We consider four three-level atoms, each of which is embedded in a separated cavity. The two long-lived states of the atom  $|0\rangle$  and  $|1\rangle$  are used as a qubit, where only the state  $|1\rangle$  is coupled to the excited state  $|e\rangle$ , whose transition frequency is equal to that of the cavity mode (see Fig. 1). The output fields of the cavities are mixed with 50:50

beam splitters (BS's) and measured by photodetectors. The effective Hamiltonian of the system is given by

$$H = \sum_{i=1}^4 \frac{g_i}{2} (|1\rangle_{ii} \langle e| \hat{c}_i^\dagger + \text{H.c.}) - i \sum_{i=1}^4 \kappa_i \hat{c}_i^\dagger \hat{c}_i,$$

where  $g_i$  denotes the coupling between the  $|1\rangle_i \leftrightarrow |e\rangle_i$  transition and the  $i$ th cavity mode  $\hat{c}_i$ . The cavity photon leaks to the output mode with rate  $2\kappa_i$  ( $\kappa_i > g_i$ ), which is treated as the non-Hermitian term by following the quantum jump approach [22]. For simplicity, the cavity parameters are set to  $g_i = g$  and  $\kappa_i = \kappa$  ( $i = 1, 2, 3, 4$ ). As shown in Fig. 1, the output modes are mixed by using the four 50:50 BS's. Thus the four detector modes  $\hat{a}_i$  are given in terms of the cavity modes  $\hat{c}_i$  by

$$\hat{a}_1 = (\hat{c}_1 + \hat{c}_2 + \hat{c}_3 + \hat{c}_4)/2, \quad \hat{a}_2 = (\hat{c}_1 - \hat{c}_2 + \hat{c}_3 - \hat{c}_4)/2, \\ \hat{a}_3 = (\hat{c}_1 + \hat{c}_2 - \hat{c}_3 - \hat{c}_4)/2, \quad \hat{a}_4 = (\hat{c}_1 - \hat{c}_2 - \hat{c}_3 + \hat{c}_4)/2.$$

The procedure to obtain the four-qubit  $W$  state  $|W_4\rangle$  is as follows. We first prepare the initial state of the atoms as  $|\Psi(0)\rangle = (|0\rangle + |e\rangle)^{\otimes 4}/4$  by using  $\pi$ -pulses. Then, we wait for a sufficiently long time  $t_w$  to detect photons. Before proceeding to the second round, each qubit is flipped as  $|0\rangle \leftrightarrow |1\rangle$ , and the state  $|1\rangle$  is excited to  $|e\rangle$  by a  $\pi$ -pulse. Then wait again for  $t_w$  to detect photons. If three and single detector clicks, or vice versa, are observed at the first and second rounds, respectively, the  $|W_4\rangle$  is obtained up to unimportant phase factors, which can be removed by using local operations.

Let us see in detail how the  $|W_4\rangle$  is generated, and calculate the success probability. For concreteness, we consider the case, where the 1st, 2nd and 3rd detectors are clicked at  $t_1, t_2, t_3$  ( $t_1 < t_2 < t_3$ ), respectively, in the first round. In the second round, the 4th detector is clicked at  $t_4$ . The state conditioned on the first three clicks is given up to normalization as

$$|\Psi(t_1, t_2, t_3)\rangle \\ = (2\kappa)^{3/2} \hat{a}_3 e^{-iH(t_3-t_2)} \hat{a}_2 e^{-iH(t_2-t_1)} \hat{a}_1 e^{-iHt_1} |\Psi(0)\rangle \\ = \frac{(2\kappa)^{3/2}}{8} \alpha(t_3) \alpha(t_2) \alpha(t_1) \left[ \mathcal{W}(|1, 0\rangle, |0, 0\rangle) \right. \\ \left. + \alpha(t_3) \mathcal{W}(|1, 0\rangle, |1, 1\rangle) + \beta(t_3) \mathcal{W}(|1, 0\rangle, |e, 0\rangle) \right],$$

where  $|a, b\rangle$  ( $a \in \{0, 1, e\}$  and  $b \in \{0, 1\}$ ) indicates the states of the atom  $|a\rangle$  and photon  $|b\rangle$ , respectively, for the combination  $\mathcal{W}(|A\rangle, |B\rangle) \equiv (|A\rangle|A\rangle|A\rangle|B\rangle - |A\rangle|A\rangle|B\rangle|A\rangle - |A\rangle|B\rangle|A\rangle|A\rangle + |B\rangle|A\rangle|A\rangle|A\rangle)/2$ . The coefficients  $\alpha(t)$  and  $\beta(t)$  are the solutions to the Schrödinger equation:

$$\alpha(t) = -ig/(2\sqrt{\kappa^2 - g^2})(-e^{\omega_+ t} + e^{\omega_- t}), \\ \beta(t) = g^2/(4\sqrt{\kappa^2 - g^2})(-e^{\omega_+ t}/\omega_+ + e^{\omega_- t}/\omega_-),$$

where  $\omega_{\pm} = (-\kappa \pm \sqrt{\kappa^2 - g^2})/2$ . The probability of such an event is given by

$$p(t_1, t_2, t_3) = |\langle \Psi(t_1, t_2, t_3) | \Psi(t_1, t_2, t_3) \rangle|^2.$$

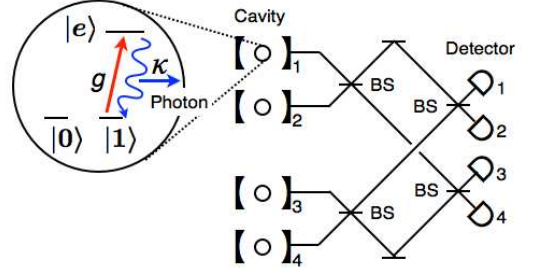


FIG. 1. Three-level atoms are embedded in cavities. The two long-lived states  $|0\rangle$  and  $|1\rangle$  are used as a qubit, and the transition between the states  $|1\rangle \leftrightarrow |e\rangle$  is coupled to the cavity mode. The output modes are mixed with four 50:50 BS's, and measured by photodetectors.

For the sufficiently long  $t_w$  ( $\gg 1/|\omega_{\pm}|$ ), the states  $|e, 0\rangle$  and  $|1, 1\rangle$  decay to  $|1, 0\rangle$  incoherently. The postmeasurement state at  $t_w$  is given by

$$\rho(t_w) = \mathcal{N} [\rho_{\mathcal{W}}(|1, 0\rangle, |0, 0\rangle) + |\alpha(t_w)|^2 |1, 0\rangle \langle 1, 0|^{\otimes 4}],$$

where  $\rho_{\mathcal{W}}(|A\rangle, |B\rangle) = \mathcal{W}(|A\rangle, |B\rangle) \mathcal{W}(|A\rangle, |B\rangle)^\dagger$ , and  $\mathcal{N} = (2\kappa)^3 |\alpha(t_3) \alpha(t_2) \alpha(t_1)|^2 / p(t_1, t_2, t_3)$ .

Before proceeding to the second round, each qubit is flipped as  $|0\rangle \leftrightarrow |1\rangle$ , and the state  $|1\rangle$  is excited to  $|e\rangle$  similarly to the first round. Then, the initial state of the second round is given by

$$\rho'(0) = \mathcal{N} [\rho_{\mathcal{W}}(|0, 0\rangle, |e, 0\rangle) + |\alpha(t_w)|^2 |0, 0\rangle \langle 0, 0|^{\otimes 4}].$$

Since the first term has exactly one excitation, by observing the single detector click at  $t_4$ , the second term is removed in this round. Finally we obtain the four-qubit  $W$  state  $|W_4\rangle$  for the atoms. The joint probability for the first three clicks and the second single click is calculated as

$$p(t_1, t_2, t_3, t_4) = p(t_4 | t_1, t_2, t_3) p(t_1, t_2, t_3) \\ = \text{Tr}[2\kappa \hat{a}_4^\dagger \hat{a}_4 e^{-iHt_4} \rho'(0) e^{iHt_4}] p(t_1, t_2, t_3) \\ = (2\kappa)^4 |\alpha(t_1) \alpha(t_2) \alpha(t_3) \alpha(t_4)|^2 / 64.$$

For  $t_w$  sufficiently long  $t_w$  ( $\gg 1/|\omega_{\pm}|$ ), the success probability is calculated as

$$\prod_{i=1}^4 \int_0^{t_w} dt_i \frac{(2\kappa)^4}{64} |\alpha(t_1) \alpha(t_2) \alpha(t_3) \alpha(t_4)|^2 = \frac{1}{64},$$

where the sum over the orderings of  $t_1, t_2, t_3$  are also taken. The number of the events with three and single detector clicks, or vice versa, at the first and second round, respectively, is  $2 \times 4 \times 4$ . Thus the total success probability is unexpectedly high to be  $p = 1/2$ . This success probability can also be understood by the fact that the initial state  $(|0\rangle + |1\rangle)^{\otimes 4}/4$  contains two types of the  $W$  states (i.e.,  $|0001\rangle \dots$  and  $|1110\rangle \dots$ ) with each probability  $1/4$ . Then in the present setup, we can fully extract the  $W$  states by virtue of the highly symmetric

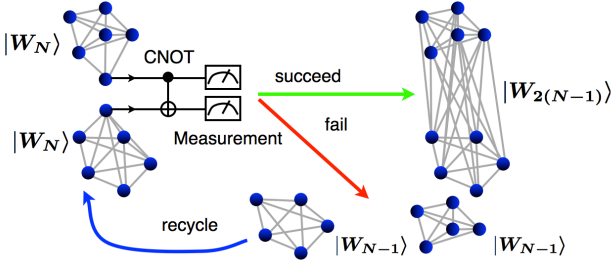


FIG. 2. Economical breeding. Two  $|W_N\rangle$ 's are converted to a  $|W_{2(N-1)}\rangle$  probabilistically. Even if the conversion fails, the two  $|W_{N-1}\rangle$ 's are left and can be recycled.

detector modes. This method inherits the robustness of the Barrett-Kok scheme [6]; the detector inefficiency and photon loss do not deteriorate the fidelity, but only decrease the success probability. The success probability scales like  $p = (\eta_d \eta_l)^4 / 2$ , where  $\eta_d$  and  $1 - \eta_l$  denote the detector efficiency and photon loss rate, respectively. Other imperfections such as decoherence of the qubits, detector dark counts, and mode mismatches would not deteriorate the fidelity crucially for a specific physical system such as the NV-diamond system, as discussed in Ref. [6].

The above process to prepare the  $|W_4\rangle$  is viewed as a single concatenation of entangling operation,  $|1\rangle \rightarrow |10\rangle + |01\rangle$  and  $|0\rangle \rightarrow |00\rangle$ . It can be extended straightforwardly to generate an  $N$ -qubit ( $N = 2^L$  with an integer  $L$ )  $W$  state  $|W_N\rangle$  with probability  $N/2^{N-1}$  by using a similar setup. The detector modes are given by  $\hat{a}_j^{(L)} = A_{ij}^{(L)} \hat{c}_i / \sqrt{N}$  in terms of an  $N \times N$  matrix  $A^{(L)}$  generated recursively by

$$A^{(L+1)} = \begin{pmatrix} A^{(L)} & A^{(L)} \\ A^{(L)} & -A^{(L)} \end{pmatrix},$$

where  $A^{(0)} = 1$ . Then single and  $N - 1$  clicks, or vice versa, at the first and second rounds, respectively, result in the  $|W_N\rangle$ . More generally, if we observe  $m$  and  $N - m$  clicks at the first and second rounds, respectively, we can obtain the Dicke-symmetric state [21]:

$$|D_{m,N-m}\rangle = \sum_i \mathcal{S}_i(|0\rangle^{\otimes m} |1\rangle^{\otimes N-m}) / \sqrt{C_{N-m}^m},$$

where  $\{\mathcal{S}_i\}$  denotes the set of all distinct combinations of the qubits, and  $C_{N-m}^m = N! / [m!(N-m)!]$ . With the increasing number of qubits  $N$ , however, the success probability  $\sim 2^{-O(N)}$  diminishes exponentially.

*Economical breeding.*— We next show that the four-qubit  $W$  states are sufficient to generate an arbitrarily large  $W$  state with a quasi-polynomial overhead, introducing an economical breeding (Fig. 2). Suppose that we have obtained the  $N$ -qubit  $W$  states:

$$|W_N\rangle = \frac{1}{\sqrt{N}} |1\rangle^{(a)} |0_{N-1}\rangle + \sqrt{\frac{N-1}{N}} |0\rangle^{(a)} |W_{N-1}\rangle,$$

where the qubit labeled by (a) is used as an ancilla for the breeding, and  $|0_n\rangle \equiv |0\rangle^{\otimes n}$ . Then, the two  $N$ -qubit  $W$  states can be rewritten as

$$\begin{aligned} & |W_N\rangle |W_N\rangle \\ &= \frac{1}{N} |11\rangle^{(a)} |0_{2(N-1)}\rangle + \frac{\sqrt{N-1}}{N} |10\rangle^{(a)} |W_{N-1}\rangle |0_{N-1}\rangle \\ &\quad + \frac{\sqrt{N-1}}{N} |01\rangle^{(a)} |0_{N-1}\rangle |W_{N-1}\rangle \\ &\quad + \frac{N-1}{N} |00\rangle^{(a)} |W_{N-1}\rangle |W_{N-1}\rangle, \end{aligned}$$

where the two ancilla qubits in the  $W$  states are moved to the first two-qubit Hilbert space labeled by (a). Here, we perform a controlled-NOT (CNOT) gate between the two ancilla qubits, and measure the second ancilla qubit in the  $Z$  basis. If the measurement outcome is 1, the postmeasurement state is given by

$$\frac{1}{\sqrt{2}} (|11\rangle^{(a)} |W_{N-1}\rangle |0_{N-1}\rangle + |01\rangle^{(a)} |0_{N-1}\rangle |W_{N-1}\rangle).$$

The probability for obtaining such an outcome is  $(N-1)/N^2$ . Next, by measuring the first ancilla qubit in the  $X$  basis and performing local operations properly depending on the outcome, we can convert the two  $N$ -qubit  $W$  state to the  $2(N-1)$ -qubit  $W$  state:

$$\frac{1}{\sqrt{2}} (|W_{N-1}\rangle |0_{N-1}\rangle + |0_{N-1}\rangle |W_{N-1}\rangle) = |W_{2(N-1)}\rangle.$$

This indicates a good property of entanglement webs; a local connection produces a global web structure.

Alternatively, if the outcome of the first measurement for the second ancilla qubit is 0, we have

$$\frac{|10\rangle^{(a)} |0_{2(N-1)}\rangle + (N-1) |00\rangle^{(a)} |W_{N-1}\rangle |W_{N-1}\rangle}{\sqrt{N^2 - 2N + 2}}.$$

Then, by measuring the first ancilla qubit in the  $Z$  basis with the outcome 0, the two  $|W_{N-1}\rangle$ 's are left, which can be recycled to generate the  $|W_{2(N-2)}\rangle$ . The joint probability to obtain such outcomes (0, 0) is  $(N-1)^2/N^2$ .

Notice in the above that in order to grow the size of the  $W$  state,  $2(N-1) > N$  is required, i.e.,  $N > 3$ . Thus starting from the four-qubit  $W$  states, we can breed an arbitrarily large  $W$  state by repeating the conversion process. Not only with an even number of qubits, we can also obtain the  $W$  state with an odd number of qubits as byproducts when the conversion fails.

In the cavity-QED setup such as Fig. 1, instead of the above procedure (CNOT and measurements), the original Barrett-Kok scheme can be used to project the ancilla qubits to the subspace spanned by  $\{|10\rangle^{(a)}, |01\rangle^{(a)}\}$ . Then, if the projection is successful with probability  $(N-1)/N^2$ , the two  $|W_N\rangle$ 's are converted to the  $|W_{2(N-1)}\rangle$ . In the failure case, then, if the ancilla qubits (atoms) are confirmed to be in the  $|00\rangle^{(a)}$  by measuring them directly, the two  $|W_{N-1}\rangle$ 's are left for recycling. Even when the

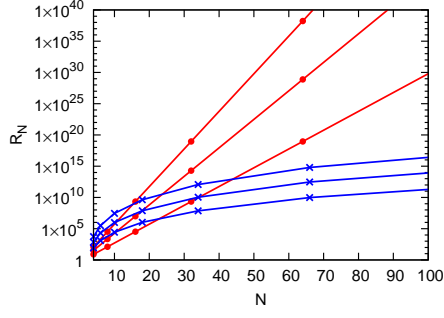


FIG. 3. The overheads  $R_N$  for the concatenated entangling (red  $\circ$ ) and the breeding (blue  $\times$ ), respectively, are plotted as functions of the number of qubits  $N$ , where  $\eta_d \eta_l = 0.5, 0.7, 1$  from top to bottom.

detector inefficiency and photon loss are considered, the conversion probability is diminished by only  $(\eta_d \eta_l)^2$ .

The success probability for the breeding sequence  $|W_4\rangle \rightarrow \cdots |W_{N_k}\rangle \rightarrow \cdots |W_N\rangle$  is calculated ( $\eta_d \eta_l = 1$  in the ideal case) as

$$p_N = \frac{1}{2} \prod_{k=1}^K \frac{2^k + 1}{(2^k + 2)^2},$$

where  $K = \log_2(N-2) - 1$  means the number of conversions required to breed the  $|W_N\rangle$ , and  $N_k = 2^{k+1} + 2$  satisfies  $N_{k+1} = 2(N_k - 1)$ . The overhead  $R_N = 4 \times 2^K / p_N$  scales like  $2^{O[(\log_2 N)^2]}$  for  $N \gg 1$ , which is quasi-polynomial in the number of qubits  $N$ . This is because, although the success probability of the conversion  $|W_N\rangle |W_N\rangle \rightarrow |W_{2(N-1)}\rangle$  decreases as  $O(1/N)$ , the size of the  $W$  state grows exponentially with the number of conversions  $O(\log_2 N)$ . (The overhead will be somewhat improved by recycling.) On the other hand, if we generate the  $|W_N\rangle$  by the concatenated entangling with  $A^{(L)}$  as mentioned before, the overhead  $R_N \sim 2^{O(N)}$  is exponential. Furthermore, the number of total clicks in the breeding is  $4 + 2K = 3 + 2\log_2(N-2)$ . Thus the detector inefficiency  $\eta_d$  and photon loss  $1 - \eta_l$  do not upset the scalability in the breeding scheme though they require somewhat more resources. In Fig. 3, the overheads  $R_N$  for the concatenated entangling (red  $\circ$ ) and the breeding (blue  $\times$ ), respectively, are plotted as functions of the number of qubits  $N$ , where  $\eta_d \eta_l = 0.5, 0.7, 1$  from top to bottom. As byproducts in breeding the  $|W_N\rangle$ , the  $|W_{N-2M}\rangle$  ( $1 \leq M \leq N/2 - 1$ ) can also be obtained with probability  $(N - 2M)p_N/N$  and resources  $4 \times 2^K [N/(N - 2M)]/p_N$  by recycling.

*Discussion and conclusion.*— We have considered a robust and scalable scheme to generate large-scale entanglement webs. We have first introduced an efficient way to generate the four-qubit  $W$  state by following the Barrett-Kok's concept, which provides a significantly high success probability  $1/2$ . Then, by using the four-qubit  $W$  states as seeds, we have developed an economical breed-

ing method to generate an arbitrarily large  $W$  state with a quasi-polynomial overhead. The breeding method is quite simple, and exploits an unique property of entanglement webs. That is, a global web structure can be constructed only by a local connection. This will provide a new perspective on multipartite entanglement.

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